Direct Design Formulas for Asymmetric Bandpass Channel Diplexers

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Abstract—**This paper gives direct design formulas for asymmetric bandpass channel diplexers, which allow rapid design of diplexers using narrow-band direct-coupled resonator filters. Computed results for a prototype diplexer are given, and results are presented for a 5.8-GHz asymmetric waveguide diplexer, which demonstrate the high performance possible using this design technique.**

I. Introduction

THIS PAPER gives direct design formulas for asymmetric bandpass channel prototype diplexers, which can be transformed into narrow-band diplexers using direct-coupled resonator filters. These formulas are a generalization of ones given for the symmetrical case in an earlier paper by Rhodes [1].

The asymmetric bandpass diplexer consists of two dissimilar bandpass filters connected in series. To compensate for the interaction between the filters, each is internally modified, and a series annulling reactance is introduced. The design procedure forces the reflection coefficient at the common port to be approximately zero at a finite set of frequencies in each channel. The reflection coefficient at each of the other ports is automatically forced to be zero to the same degree of approximation at the set of frequencies in the corresponding channel.

Compared to the filters operating in isolation, each channel of the diplexer shows a significant increase in skirt selectivity over the passband of the other, which may allow a reduction of degree to meet a given specification. The return loss at each port is only slightly degraded, and this may be allowed for by improving the specification of the original bandpass filters.

Section II quotes the formulas without proof: the proof is given in Section III. Section IV gives computed responses for a typical prototype diplexer, and proceeds to consider the design of a waveguide diplexer at 5.8 GHz. The measured response of this diplexer agrees closely with theory.

II. The Design Formulas

The essentials of the type of diplexer considered here are shown in Fig. 1. It consists of a pair of bandpass filters, whose input impedance in the stopband tends to a short circuit, connected in series. The input impedance at the common port of the diplexer approximates to a constant resistance in the passband of each filter. Since the stopband

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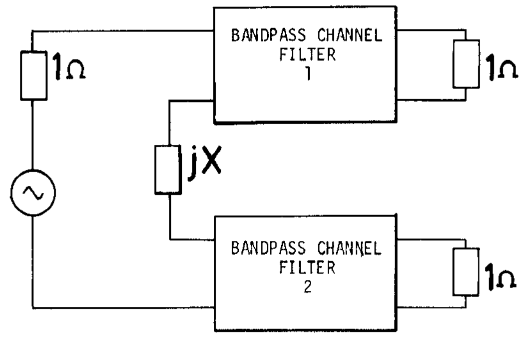


Fig. 1. An elementary series connected diplexer.

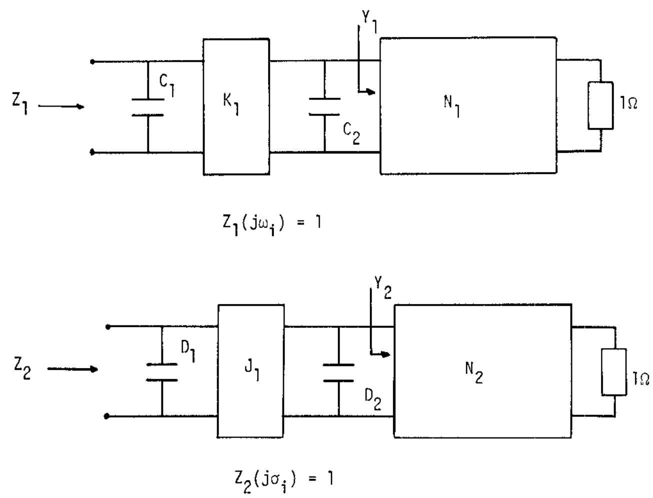


Fig. 2. Low-pass prototype networks used in the design procedure.

input impedance of each filter is actually finite and complex, the filters interact unless their passbands are very widely separated. To compensate for this interaction, each filter is modified internally, and the frequency-invariant annulling reactance X_0 is connected in series with the filters.

Let the bandpass filters be based on the normalized, doubly terminated low-pass prototype networks shown in Fig. 2. Each prototype begins with a pair of capacitors coupled through an admittance inverter, the rest of the network being arbitrary. The derivation of the design formulas requires that the input impedance of each prototype be unity at some set of frequencies in the passband, and each filter must have at least a second-ordered transmission zero at infinity. Since the networks are assumed lossless, the first condition implies that they have zero insertion loss at the same sets of frequencies. Prototype element values for Chebyshev, Butterworth, or even linear-phase filters are available from explicit formulas or fairly simple synthesis procedures [2].

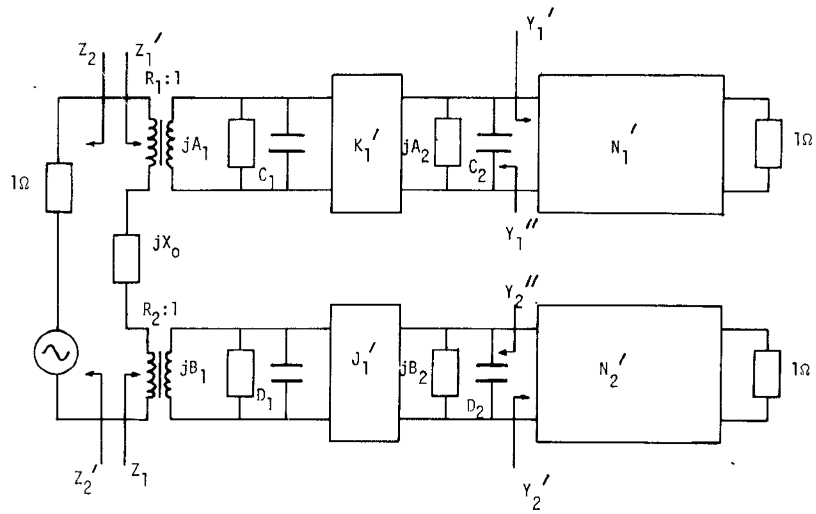


Fig. 3. The prototype diplexer circuit.

Fig. 3 shows the diplexer prototype circuit. The channels have been transformed to bandpass filters at center frequencies $\pm\alpha$. The section of each prototype following the first two shunt capacitors is transformed by the direct frequency transformation:

$$\omega \rightarrow \omega \mp \alpha. \tag{1}$$

The elements in the first two "resonators" of each filter are obtained from the formulas (2)–(10) below, which are equivalent to (1) as $\alpha \rightarrow \infty$. If the prototypes are identical, these formulas reduce to those given in [1].

The approximate improvements in the insertion loss of each channel in the center of the passband of the other are given in (11) and (12). ΔL_u is the increase in insertion loss of the upper filter in the center of the passband of the lower filter, compared to the filter in isolation, and correspondingly for ΔL_L.

Referring to Fig. 3, the design formulas are as follows:

$$X_0 = \frac{1}{2\alpha}\left(\frac{1}{D_1} - \frac{1}{C_1}\right) \tag{2}$$

$$A_1 = -C_1\left(\alpha + \frac{1}{2C_1^2\alpha} + \frac{1}{8D_1^2C_1\alpha^3}\left[\frac{J_1^2}{D_2} - \frac{1}{C_1}\right]\right) \tag{3}$$

$$A_2 = -C_2\left(\alpha + \frac{K_1^2}{8C_1^2C_2D_1\alpha^3}\right) \tag{4}$$

$$B_1 = D_1\left(\alpha + \frac{1}{2D_1^2\alpha} + \frac{1}{8C_1^2D_1\alpha^3}\left[\frac{K_1^2}{C_2} - \frac{1}{D_1}\right]\right) \tag{5}$$

$$B_2 = D_2\left(\alpha + \frac{J_1^2}{8D_1^2D_2C_1\alpha^3}\right) \tag{6}$$

$$R_1^2 = 1 + \frac{1}{4C_1\alpha^2}\left(\frac{1}{C_1} - \frac{1}{D_1}\right) \tag{7}$$

$$R_2^2 = 1 + \frac{1}{4D_1\alpha^2}\left(\frac{1}{D_1} - \frac{1}{C_1}\right) \tag{8}$$

$$K_1'^2 = K_1^2\left(1 - \frac{1}{4C_1D_1\alpha^2}\right) \tag{9}$$

$$J_1'^2 = J_1^2\left(1 - \frac{1}{4C_1D_1\alpha^2}\right) \tag{10}$$

$$\Delta L_u = 6 + 10\log_{10}\left(1 + \frac{1}{4C_1^2\alpha^2}\right)\text{dB} \tag{11}$$

$$\Delta L_L = 6 + 10\log_{10}\left(1 + \frac{1}{4D_1^2\alpha^2}\right)\text{dB.} \tag{12}$$

III. Proof of the Design Formulas

The element values in (2)–(10) are given in terms of the band separation factor α. The transformation $\alpha \rightarrow -\alpha$ should clearly interchange the channels, and the further transformation $\omega \rightarrow -\omega$ will interchange them again. Thus application of both transformations must result in a network which is indistinguishable at its ports from the original. Considering the affects which these transformations have on the frequency invariant susceptances of the original network, these susceptances must be odd functions of α. Similarly, the transformers and admittance inverters must be even functions of α.

As the separation between the channels becomes very large, the interactions between them become vanishingly small; thus, writing the element values as power series in α^{-1}, they must be of the following form:

$$X_0 = \frac{x_1}{\alpha} + \frac{x_3}{\alpha^3} + \varepsilon(\alpha^{-5}) \tag{13}$$

$$A_1 = -C_1\left(\alpha + \frac{a_{11}}{\alpha} + \frac{a_{13}}{\alpha^3} + \varepsilon(\alpha^{-5})\right) \tag{14}$$

$$A_2 = -C_2\left(\alpha + \frac{a_{21}}{\alpha} + \frac{a_{23}}{\alpha^3} + \varepsilon(\alpha^{-5})\right) \tag{15}$$

$$B_1 = D_1\left(\alpha + \frac{b_{11}}{\alpha} + \frac{b_{13}}{\alpha^3} + \varepsilon(\alpha^{-5})\right) \quad (16)$$

$$B_2 = D_2\left(\alpha + \frac{b_{21}}{\alpha} + \frac{b_{23}}{\alpha^3} + \varepsilon(\alpha^{-5})\right) \quad (17)$$

$$R_1^2 = 1 + \frac{r_1}{\alpha^2} + \varepsilon(\alpha^{-4}) \quad (18)$$

$$R_2^2 = 1 + \frac{r_2}{\alpha^2} + \varepsilon(\alpha^{-4}) \quad (19)$$

$$K_1'^2 = K_1^2\left(1 + \frac{k_1}{\alpha^2} + \varepsilon(\alpha^{-4})\right) \quad (20)$$

$$K_2'^2 = K_2^2\left(1 + \frac{k_2}{\alpha^2} + \varepsilon(\alpha^{-4})\right). \quad (21)$$

In these equations, $\varepsilon(-)$ means "an error of the order of $(-)$."

All the other elements in the network are obtained from the direct transformation (1), and thus only the first two resonators in each filter are modified from the values they have when the filter is working in isolation.

Multiplying (24a) and (24b) by the denominator of (24a), and equating terms of equal degree in α, we get

$$2jD_1 z_1 = 1, \qquad z_1 = \frac{-j}{2D_1}$$

$$\frac{\omega_i}{2} + 2jD_1 z_2 = 0, \qquad z_2 = \frac{j\omega_i}{4D_1}$$

$$\frac{1}{2D_1}\left(D_1 b_{11} - \frac{J_1^2}{2D_2}\right) - \frac{\omega_i^2}{4} + 2jD_1 z_3 = r_2,$$

$$z_3 = \frac{j}{2D_1}\left(\frac{b_{11}}{2} - \frac{J_1^2}{4D_1D_2} - \frac{\omega_i^2}{4} - r_2\right).$$

Thus

$$Z_2(j(\omega_i + \alpha)) = 1 + \frac{j}{\alpha}\left(x_1 - \frac{1}{2D_1}\right) + j\frac{\omega_i}{4D_1\alpha^2} + \frac{j}{\alpha^3}\left[x_3 + \frac{1}{2D_1}\left(\frac{b_{11}}{2} - \frac{J_1^2}{4D_1D_2} - \frac{\omega_i^2}{4} - r_2\right)\right] + \varepsilon(\alpha^{-4}). \quad (25)$$

Knowing Z_2, we can calculate $Y_1''(j(\omega_i + \alpha))$:

$$Y_1''(j(\omega_i + \alpha)) = j(\omega_i + \alpha)C_2 - jC_2(\alpha + a_{21}/\alpha + a_{23}/\alpha^3) + \frac{K_1^2(1 + k_1/\alpha^2)}{j(\omega_i + \alpha)C_1 - jC_1(\alpha + a_{11}/\alpha + a_{13}/\alpha^3) + \dfrac{(1 + r_1/\alpha^2)}{1 + z_1/\alpha + z_2/\alpha^2 + z_3/\alpha^3}}. \quad (26)$$

Let the prototype filters of Fig. 2 have an input impedance of unity, and, hence, zero insertion loss, at the sets of frequencies $\{\omega_i\}$, $\{\sigma_i\}$. Since there is maximum power transfer through the filter at these frequencies, it follows that, at these frequencies, the input admittances $Y_1(j\omega_i)$ and $Y_2(j\sigma_i)$ are a conjugate match to the admittances looking towards the first two elements of each filter. After applying the frequency transformation $\omega \to \omega \mp \alpha$ to the subnetworks N_1 and N_2 to get N_1' and N_2', respectively (Fig. 3), it follows that

$$Y_1'(j(\omega_i + \alpha)) = -j\omega_i C_2 + \frac{K_1^2}{-j\omega_i C_1 + 1} \quad (22)$$

$$Y_2'(j(\sigma_i - \alpha)) = -j\sigma_i D_2 + \frac{J_1^2}{-j\sigma_i D_i + 1}. \quad (23)$$

We may now derive a power series expansion of the impedance $Z_1(j(\omega_i + \alpha))$, which is the input impedance of the lower filter in the passband of the upper filter:

$$Z_1(j(\omega_i + \alpha)) = \frac{1 + r_2/\alpha^2}{j(\omega_i + \alpha)D_1 + jD_1\left(\alpha + \dfrac{b_{11}}{\alpha} + \dfrac{b_{13}}{\alpha^3}\right) + \dfrac{J_1^2(1 + k_2/\alpha^2)}{j(\omega_i + \alpha)D_2 + jD_2(\alpha + \cdots)}}$$

$$= \frac{1 + r_2/\alpha^2}{2j\alpha D_1 + j\omega_i D_1 + \dfrac{j}{\alpha}\left(D_1 b_{11} - \dfrac{J_1^2}{2D_2}\right) + \varepsilon(\alpha^{-2})} \quad (24a)$$

$$= \frac{z_1}{\alpha} + \frac{z_2}{\alpha^2} + \frac{z_3}{\alpha^3} + \varepsilon(\alpha^{-4}). \quad (24b)$$

Now (26) can be expanded as a power series in α^{-1}. Letting $\alpha \to \infty$ in (26), we obtain the leading term of the power series, which is

$$Y_1'' = j\omega_i C_2 + \frac{K_1^2}{j\omega_i C_i + 1}$$

a conjugate match for Y_1', (22). We must therefore choose the unknowns in (13)–(21) so that the higher order terms in the expansion of Y_1'' are zero: this is possible up to the third degree in α. Before going on to consider the equations in the unknowns that this gives, note that similar equations can be obtained for the upper filter in the passband of the lower filter, at the set of frequencies $\{\sigma_i - \alpha\}$, to force similar conditions on Y_2''.

Expanding Y_1'' and Y_2'' and equating the coefficients of the first-, second-, and third-degree terms in α^{-1} to zero we get the following equations: from the α^{-1} terms:

$$-a_{21}+\frac{1}{2D_1}-x_1-C_1a_{11}=0 \qquad (27a)$$

$$b_{21}+D_1b_{11}-x_1-\frac{1}{2C_1}=0 \qquad (27b)$$

from the α^{-2} terms:

$$k_1(1+j\omega_iC_1)-\left[r_1-\left(x_1-\frac{1}{2D_1}\right)^2-\frac{j\omega_i}{4D_1}\right]=0 \qquad (28a)$$

$$k_2(1+j\sigma_iD_1)-\left[r_2-\left(x_1+\frac{1}{2C_1}\right)^2-\frac{j\sigma_i}{4C_1}\right]=0. \qquad (28b)$$

From the ω_i dependent terms in (28a) and (28b) we get the following independent equations:

$$k_1C_1+\frac{1}{4D_1}=0 \qquad k_2D_1+\frac{1}{4C_1}=0$$

$$k_1=k_2=-\frac{1}{4C_1D_1}. \qquad (29)$$

The α^{-3} terms are more complicated:

$$-jC_2a_{23}(1+j\omega_iC_1)^2 - K_1^2\left\{\frac{\omega_i}{4D_1}\left(\frac{1}{2D_1}-x_1\right)-j\left(x_1-\frac{1}{2D_1}\right)\cdot\left(r_1-\left[x_1-\frac{1}{2D_1}\right]^2-j\frac{\omega_i}{4D_1}\right) -j\left(x_3+\frac{1}{2D_1}\left[\frac{b_{11}}{2}-\frac{J_1^2}{4D_1D_2}-\frac{\omega_i^2}{4}-r_2\right]\right)\right\} -jC_1a_{13}=0 \qquad (30a)$$

$$jD_2b_{23}(1+j\sigma_iD_1)^2 + J_1^2\left\{\frac{\sigma_i}{4C_1}\left(x_1+\frac{1}{2C_1}\right)+j\left(x_1+\frac{1}{2C_1}\right)\cdot\left(r_2-\left[x_1+\frac{1}{2C_1}\right]^2-j\frac{\sigma_i}{4C_1}\right) +j\left(x_3-\frac{1}{2C_1}\left[\frac{a_{11}}{2}-\frac{K_1^2}{4C_1C_2}-\frac{\sigma_i^2}{4}-r_1\right]\right)\right\} -jD_1b_{13}=0. \qquad (30b)$$

The ω_i^2 term from (30a) gives

$$jC_1^2C_2a_{23}-j\frac{K_1^2}{8D_1}=0, \qquad a_{23}=\frac{K_1^2}{8C_1^2C_2D_1} \qquad (31)$$

and the σ_i^2 term from (30b) gives

$$-jD_1^2D_2b_{23}+j\frac{J_1^2}{8C_1}=0, \qquad b_{23}=\frac{J_1^2}{8D_1^2D_2C_1}. \qquad (32)$$

Substituting into (30a) and (30b) and solving for the ω_i terms gives, from (30a),

$$\frac{1}{2C_1}-\frac{1}{2D_1}+x_1=0, \qquad x_1=\frac{1}{2}\left(\frac{1}{D_1}-\frac{1}{C_1}\right) \qquad (33)$$

and this result is also given by (30b).

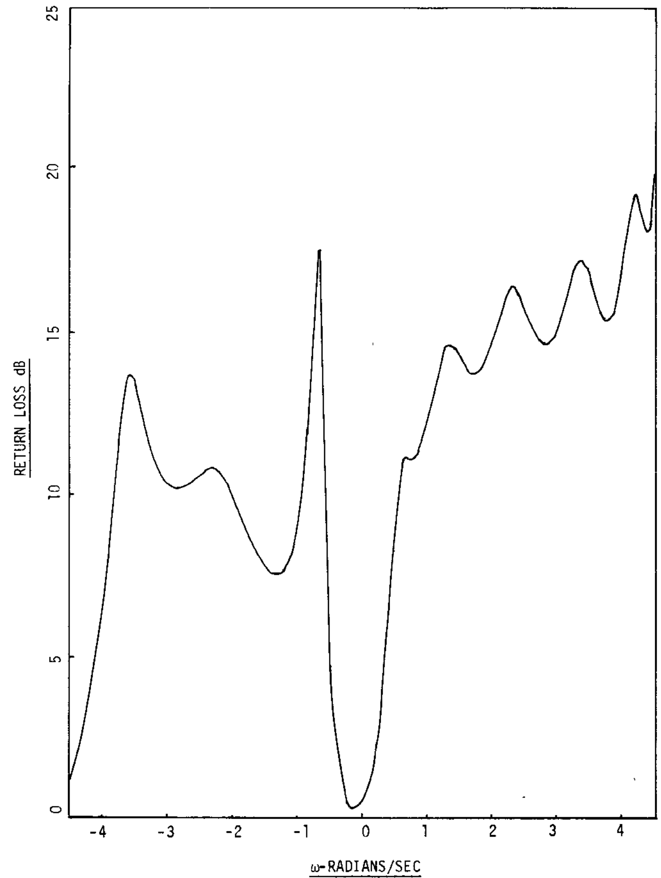


Fig. 4. Common-port return loss of unmodified filters connected in series.

Substituting for x_1 in (27a) and (27b), both a_{21} and b_{21} can be zero with no loss of generality, and we obtain

$$a_{11}=\frac{1}{2C_1^2} \qquad (34)$$

$$b_{11}=\frac{1}{2D_1^2}. \qquad (35)$$

Eliminating x_1, k_1, and k_2 in (28a) and (28b), we get

$$r_1=\frac{1}{4C_1}\left(\frac{1}{C_1}-\frac{1}{D_1}\right) \qquad (36)$$

$$r_2=\frac{1}{4D_1}\left(\frac{1}{D_1}-\frac{1}{C_1}\right). \qquad (37)$$

Finally, substituting into (30a) and (30b) and solving for the constant term, we find that x_3 can be zero without loss of generality, and

$$a_{13}=\frac{1}{8D_1^2C_1}\left(\frac{J_1^2}{D_2}-\frac{1}{C_1}\right) \qquad (38)$$

$$b_{13}=\frac{1}{8C_1^2D_1}\left(\frac{K_1^2}{C_2}-\frac{1}{D_1}\right). \qquad (39)$$

Equations (29) and (31)–(39) are the coefficients in (2)–(10).

Consider the upper filter in Fig. 3. The design procedure forces

$$Y_1''(j(\omega_1+\alpha))=Y_1'^*(j(\omega_i+\alpha))+\varepsilon(\alpha^{-4})$$

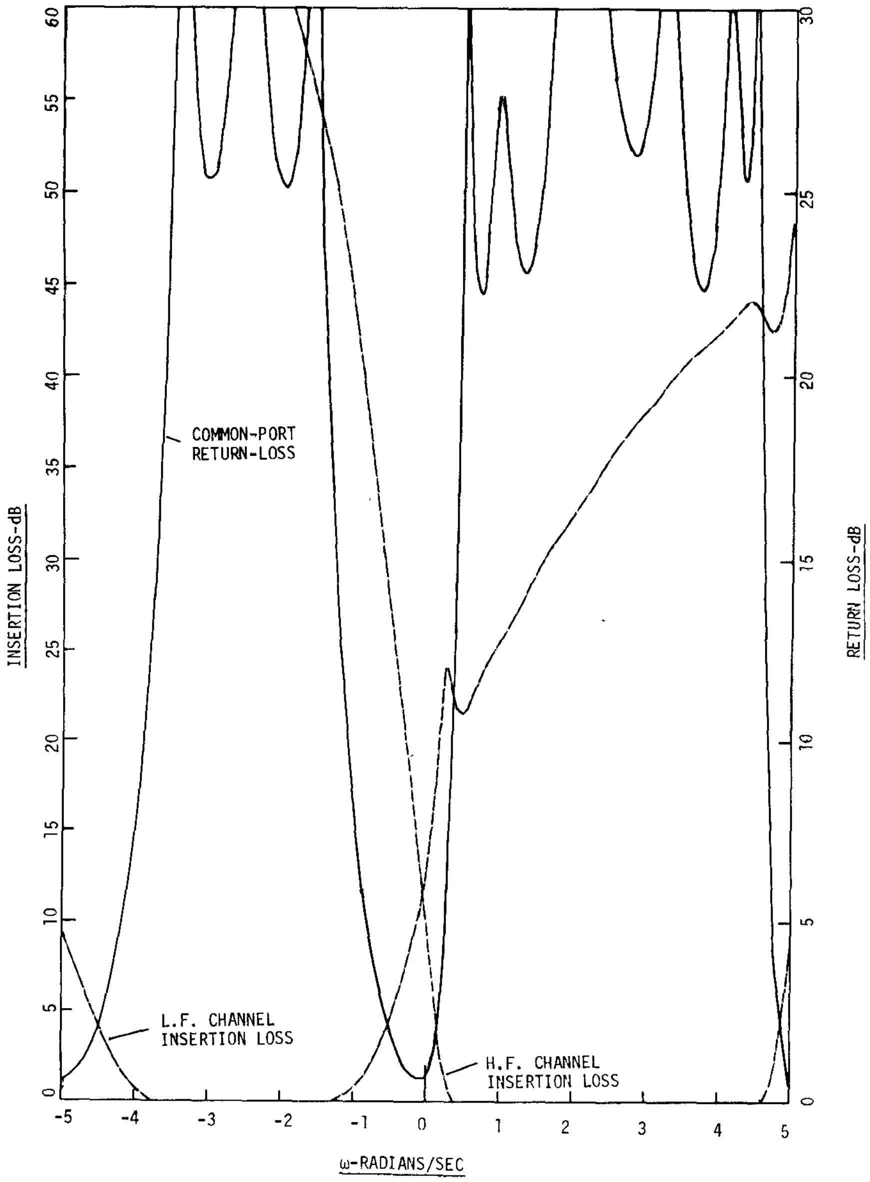


Fig. 5. Response of the prototype diplexer example.

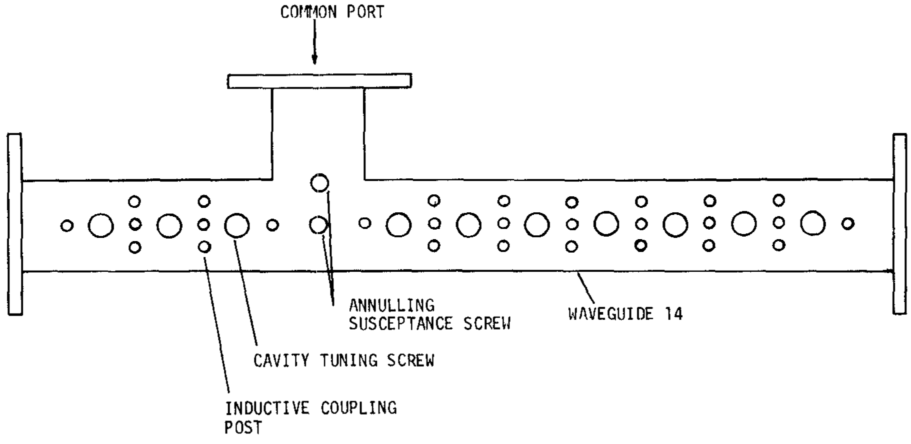
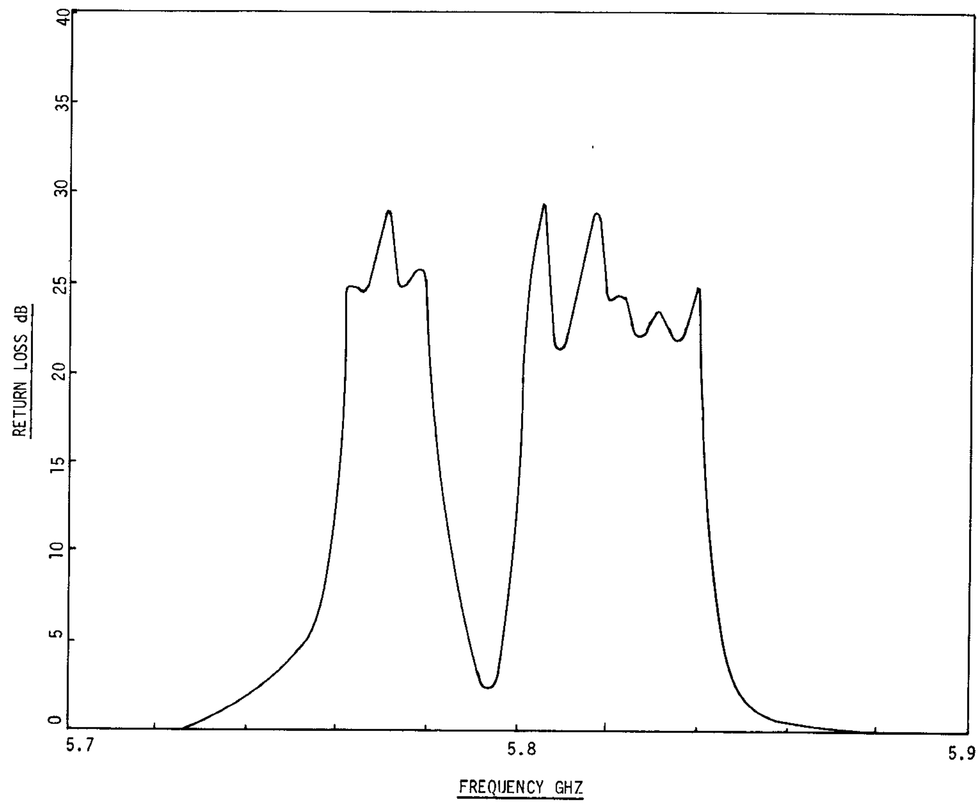
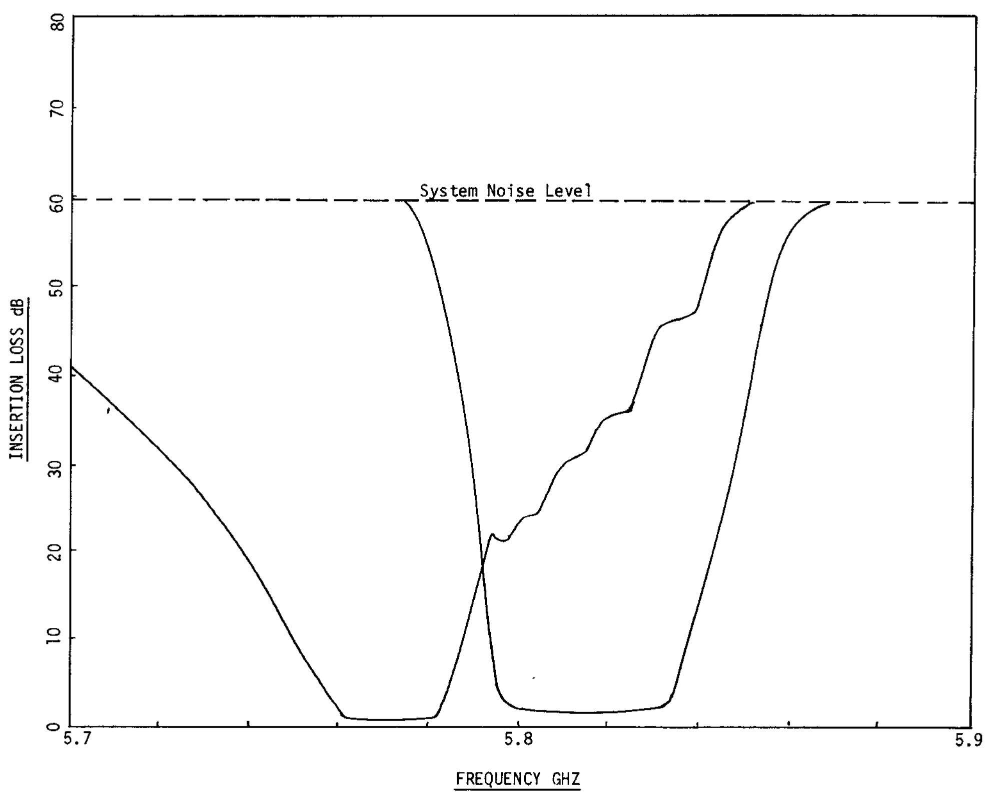


Fig. 6. Diplexer in waveguide 14.



(a)



(b)

Fig. 7. Response of the waveguide diplexer. (a) Return loss. (b) Insertion loss.

and, except for an error of the order of α^{-4}, there must therefore be maximum power transfer into N'_1. Since the lower filter, in its stopband, has an input impedance which is purely reactive up to the order of α^{-4}, it follows that, to this order of approximation, there is maximum power transfer from the source to the whole network. Also, since N'_1 is lossless, there is maximum power transfer from N'_1 to its load. Thus it follows that

$$S_{11}(j(\omega_i + \alpha)) = 0 + \varepsilon(\alpha^{-4})$$

$$S_{22}(j(\omega_i + \alpha)) = 0 + \varepsilon(\alpha^{-4})$$

and similar conditions hold for the common port and the output port of the other channel at the set of frequencies $\{\sigma_i - \alpha\}$.

Now consider the upper filter of Fig. 3 at the center of the passband of the lower filter. The increase in the insertion loss of the filter is due to the potential-divider action which results from the passband input impedance of the lower filter and the annulling reactance being connected in the series with the source resistance. Considering the modifications to the filters, it is easy to show that the increase in insertion loss is given by

$$\Delta L_u \approx 6 + 10 \log_{10}\left(1 + \frac{1}{4C_1^2\alpha^2}\right) \tag{40}$$

and similarly

$$\Delta L_H \approx 6 + 10 \log_{10}\left(1 + \frac{1}{4D_1^2\alpha^2}\right). \tag{41}$$

This process can be continued, and corrections of the fourth and higher degrees in $1/\alpha$ obtained. However, the magnitude of the corrections becomes small compared with manufacturing tolerances, and the principle corrections are to the resonant frequencies of the elements and are inevitably made during tuning of the diplexer.

In [1], Rhodes showed that the symmetrical diplexer performed well even when the channels were contiguous. This is not true of the asymmetric diplexer, unless the selectivities of the individual filters are comparable at their band edges, and this criterion must be used to determine the filter specification for a given application.

IV. Performance of a Typical Diplexer

Consider the following diplexer example.

	Center Frequency (rad/s)	Bandwidth (rad/s)	Return Loss ripple (dB)	Degree
Low-Frequency Channel	−2.5	2.0	26	3
High-Frequency Channel	+2.5	4.0	27.31	7

Each filter has a conventional Chebyshev response and the explicit element values given in [2]. The degree and ripple level have been chosen such that the insertion losses of each channel are equal at zero frequency.

The common-port return-loss response of the two filters connected directly in series is shown in Fig. 4; clearly, the interactions between the filters cause an unacceptable degradation of performance.

In contrast, Fig. 5 shows the performance obtained by modifying the filters according to the design procedure given in Section II: the improvement over Fig. 4 is very marked. The in-channel return loss is better than 22 dB, and the lower frequency channel shows very little degradation. The insertion-loss curves also show the predicted improvement.

A waveguide diplexer has been designed from this prototype. This was made in waveguide 14, at a center frequency of 5.8 GHz and channel bandwidths of 20 and 40 MHz; the channel separation is hence 50 MHz. The design of the diplexer was straightforward, and is sketched in Fig. 6. The filters were designed using direct-coupled half-wave cavities, coupled via shunt inductive post irises. The individual filters were designed using the procedure in [3], and the common port was realized by a T-junction with tuning screws at the center of the junction to provide a capacitive annulling susceptance. The frequency-invariant susceptances in parallel with the capacitors in the prototype produce relative differences in the cavity resonant frequencies and were absorbed by adjustments to the tuning screws.

The response of the waveguide diplexer agreed closely with theory, as is shown in Fig. 7.

V. Conclusions

The direct design formulas given for asymmetric bandpass channel diplexers can be used to design high-performance diplexer prototypes. These prototypes can be directly transformed into narrow-band diplexers using direct-coupled resonator filters.

A 5.8-GHz waveguide diplexer has been designed and built using the new formulas, and its response agreed closely with theory.

References

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